MATIČNI BROJ STUDENTA | IME I PREZIME | BROJ BODOVA

LINEARNA ALGEBRA 1

1. kolokvij - 23. studenog 2016.

ZADATAK 3

Zadana je matrica $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$ i potprostori K, L, N vektorskog prostora $M_2(\mathbb{R})$:

$$K = [\{A^t, A+A^t, (A+I)A\}], \quad L = \{X \in M_2(\mathbb{R}) : AX = 0\}, \quad N = \{X \in M_2(\mathbb{R}) : XA = A^tX\},$$

gdje je s A^t označena transponirana matrica matrice A, a s I jedinična matrica.
Nađite neku bazu potprostora $(K \cap L) + N$.

$$K = [\{ \underbrace{\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}}_{A^t}, \underbrace{\begin{bmatrix} 4 & -3 \\ -3 & -4 \end{bmatrix}}_{A+A^t}, \underbrace{\begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}}_{\text{izbacimo } A} \}] = [\{ \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 4 & -3 \\ -3 & -4 \end{bmatrix} \}]$$

$$(A+I)A = A^2 + IA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + A = A$$

$$L = \{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : \begin{bmatrix} 2a+c & 2b+d \\ -4a-2c & -4b-2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \} = \{ \begin{bmatrix} a & b \\ -2a & -2b \end{bmatrix} : a, b \in \mathbb{R} \} = [\{ \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \}]$$

$$\left.\begin{aligned} 2a+c &= 0 \\ 2b+d &= 0 \end{aligned}\right\} \Leftrightarrow \begin{aligned} c &= -2a \\ d &= -2b \end{aligned}$$

$$K + L = [\{ \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 4 & -3 \\ -3 & -4 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \}]$$

$$\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = \alpha \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} + \beta \begin{bmatrix} 4 & -3 \\ -3 & -4 \end{bmatrix} \Leftrightarrow \left\{ \begin{aligned} 2\alpha + 4\beta &= 1 \\ -4\alpha - 3\beta &= 0 \\ \alpha - 3\beta &= -2 \\ -2\alpha - 4\beta &= 0 \end{aligned} \right\} \alpha = \beta = 0 \Rightarrow\Leftarrow$$

$$\begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} = \underbrace{\alpha \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} + \beta \begin{bmatrix} 4 & -3 \\ -3 & -4 \end{bmatrix}}_{K\text{-dio}} + \underbrace{\gamma \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}}_{L \text{ dio}} \Leftrightarrow \left\{ \begin{aligned} 2\alpha + 4\beta + \gamma &= 0 \\ -4\alpha - 3\beta &= 1 \\ \alpha - 3\beta - 2\gamma &= 0 \\ -2\alpha - 4\beta &= -2 \end{aligned} \right\} \begin{aligned} \beta &= 1 \\ \alpha &= -1 \\ \gamma &= -2 \end{aligned}$$

$$K \cap L = [\{ -\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} + \begin{bmatrix} 4 & -3 \\ -3 & -4 \end{bmatrix} \}] = [\{ \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} \}] = [\{A\}]$$

$$N = \{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : \begin{bmatrix} 2a-4b & a-2b \\ 2c-4d & c-2d \end{bmatrix} = \begin{bmatrix} 2a-4c & 2b-4d \\ a-2c & b-2d \end{bmatrix} \} = \{ \begin{bmatrix} 4b-4d & b \\ b & d \end{bmatrix} : b, d \in \mathbb{R} \}$$

$$= [\{ \begin{bmatrix} 4 & 1 \\ 1 & 0 \end{bmatrix}, \underbrace{\begin{bmatrix} -4 & 0 \\ 0 & 1 \end{bmatrix}}_{\text{ili } \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}} \}]$$

$$\left.\begin{aligned} 2a-4b &= 2a-4c \\ a-2b &= 2b-4d \\ 2c-4d &= a-2c \\ c-2d &= b-2d \end{aligned}\right\} \Leftrightarrow \begin{aligned} & b = c \\ & a-4b+4d = 0 \\ & a = 4b-4d \\ & c = b \end{aligned}$$

$$(K \cap L) + N = [\underbrace{\{ \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -4 & 0 \\ 0 & 1 \end{bmatrix} \}}_{\text{baza}}]$$

jer je svaka lin. kombinacija druga dva vektora simetrična matrica.

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ZADATAK 3

$$B^t = \begin{bmatrix} -3 & -1 \\ 9 & 3 \end{bmatrix}$$

Zadana je matrica $B = \begin{bmatrix} -3 & 9 \\ -1 & 3 \end{bmatrix}$ i potprostori K, L, N vektorskog prostora $M_2(\mathbb{R})$:

$$K = \{X \in M_2(\mathbb{R}) : XB = 0\}, \quad L = [\{B, B-B^t, B(B-I)\}], \quad N = \{X \in M_2(\mathbb{R}) : XB^t = BX\},$$

gdje je s B^t označena transponirana matrica matrice B, a s I jedinična matrica.
Nađite neku bazu potprostora $K + (L \cap N)$.

$$K = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : \begin{bmatrix} -3a-b & 9a+3b \\ -3c-d & 9c+3d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} a & -3a \\ c & -3c \end{bmatrix} : a, c \in \mathbb{R} \right\} = \left[\left\{ \begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -3 \end{bmatrix} \right\}\right]$$

$$B(B-I) = B^2 - B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - B = -B$$

$$L = \left[\left\{ \underset{B}{\begin{bmatrix} -3 & 9 \\ -1 & 3 \end{bmatrix}}, \underset{B-B^t}{\begin{bmatrix} 0 & 10 \\ -10 & 0 \end{bmatrix}}, \underbrace{\begin{bmatrix} 3 & -9 \\ 1 & -3 \end{bmatrix}}_{\text{izbacimo } -B} \right\}\right] = \left[\left\{ \begin{bmatrix} -3 & 9 \\ -1 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 10 \\ -10 & 0 \end{bmatrix} \right\}\right]$$

$$N = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : \begin{bmatrix} -3a+9b & -a+3b \\ -3c+9d & -c+3d \end{bmatrix} = \begin{bmatrix} -3a+9c & -3b+9d \\ -a+3c & -b+3d \end{bmatrix} \right\} = \left\{ \begin{bmatrix} 6b-9d & b \\ b & d \end{bmatrix} : b, d \in \mathbb{R} \right\}$$

$$\left.\begin{aligned} -3a+9b &= -3a+9c \\ -a+3b &= -3b+9d \\ -3c+9d &= -a+3c \\ -c+3d &= -b+3d \end{aligned}\right\} \iff \begin{aligned} & b = c \\ & \underline{a - 6b + 9d = 0} \\ & a = 6b - 9d \\ & c = b \end{aligned} \qquad = \left[\left\{ \begin{bmatrix} 6 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -9 & 0 \\ 0 & 1 \end{bmatrix} \right\}\right]$$

$$L + N = \left[\left\{ \underbrace{\begin{bmatrix} -3 & 9 \\ -1 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 10 \\ -10 & 0 \end{bmatrix}}, \begin{bmatrix} 6 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -9 & 0 \\ 0 & 1 \end{bmatrix} \right\}\right]$$

$$\begin{bmatrix} 6 & 1 \\ 1 & 0 \end{bmatrix} = \alpha \begin{bmatrix} -3 & 9 \\ -1 & 3 \end{bmatrix} + \beta \begin{bmatrix} 0 & 10 \\ -10 & 0 \end{bmatrix} \iff \begin{cases} -3\alpha = 6 \\ 9\alpha + 10\beta = 1 \\ -\alpha - 10\beta = 1 \\ 3\alpha = 0 \end{cases} \quad \alpha = 0 \Rightarrow \Leftarrow$$

$$\begin{bmatrix} -9 & 0 \\ 0 & 1 \end{bmatrix} = \underbrace{\alpha \begin{bmatrix} -3 & 9 \\ -1 & 3 \end{bmatrix} + \beta \begin{bmatrix} 0 & 10 \\ -10 & 0 \end{bmatrix}}_{L\text{-dio}} + \underbrace{\gamma \begin{bmatrix} 6 & 1 \\ 1 & 0 \end{bmatrix}}_{N\text{-dio}} \iff \begin{cases} -3\alpha + 6\gamma = -9 \\ 9\alpha + 10\beta + \gamma = 0 \\ -\alpha - 10\beta + \gamma = 0 \\ 3\alpha = 1 \end{cases} \quad \begin{aligned} \alpha &= \tfrac{1}{3} \\ \gamma &= -\tfrac{4}{3} \\ \beta &= -\tfrac{1}{6} \end{aligned}$$

$$L \cap N = \left[\left\{ \frac{1}{3} \begin{bmatrix} -3 & 9 \\ -1 & 3 \end{bmatrix} - \frac{1}{6} \begin{bmatrix} 0 & 10 \\ -10 & 0 \end{bmatrix} \right\}\right] = \left[\left\{ \begin{bmatrix} -1 & \frac{4}{3} \\ \frac{4}{3} & 1 \end{bmatrix} \right\}\right] = \left[\left\{ \begin{bmatrix} -6 & 8 \\ 8 & 6 \end{bmatrix} \right\}\right] = [\{B + B^t\}]$$

$$K + (L \cap N) = \left[\left\{ \underbrace{\begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -3 \end{bmatrix}, \begin{bmatrix} -1 & \frac{4}{3} \\ 4/3 & 1 \end{bmatrix}}_{} \right\}\right]$$

očito je baza jer se treći vektor ne može zapisati kao linearna kombinacija prve dve