Two Difficult Problems in Extension Theory

Leonard R. Rubin

Abstract

Extension theory is based on the following notion. Given a space X and a CW-complex K, one says that K is an absolute extensor for X, written $X\tau K$, if for each closed subset A of X and map $f : A \to K$, there exists a map $F : X \to K$ that extends f. Given a class $\mathcal{C}$ of spaces and a class $\mathcal{T}$ of CW-complexes, one can define an equivalence relation on $\mathcal{T}$ so that if $K \in \mathcal{C}$, then its equivalence class is denoted $[K]_{(\mathcal{C},\mathcal{T})}$. Then for $X \in \mathcal{C}$, $X\tau K$ implies that $X\tau L$ for all $L \in [K]_{(\mathcal{C},\mathcal{T})}$. We shall describe this relation in our talk. From now on, let $\mathcal{C}$ be the class of compact metrizable spaces and $\mathcal{T}$ the class of all CW-complexes.

In this theory there is a notion of the extension dimension of a given space $X \in \mathcal{C}$, and it has been proven that there always exists $K \in \mathcal{T}$ such that the extension dimension of X equals $[K]_{(\mathcal{C},\mathcal{T})}$. The latter is a type of unique "minimal" element in a certain partially ordered class.

Problem 1. Determine whether for each $X \in \mathcal{C}$ there exists a countable CW-complex K such that the extension dimension of X equals $[K]_{(\mathcal{C},\mathcal{T})}$.

Problem 2. For a given $K \in \mathcal{T}$ determine if there exists $X \in \mathcal{C}$ such that X is universal for $[K]_{(\mathcal{C},\mathcal{T})}$. Put simply, we ask for $X \in \mathcal{C}$ such that $X\tau K$ and if $Y \in \mathcal{C}$ and $Y\tau K$, then Y embeds in X.

University of Oklahoma, Norman, Oklahoma, USA
lrubin@ou.edu