Znanstveno računanje 2
2. predavanje

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Sadržaj predavanja

- Primjer iz prakse:
  - Numerički model hlađenja u pećima za prokaljivanje plinom.

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Sadržaj

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Uvod

Original je prezentacija pod naslovom:

- "A Numerical Model of Cooling in Gas–Quenching Systems"

s konferencije

- "International Conference on Numerical Analysis and Applied Mathematics" (ICNAAM 2005),
- Hotel Esperides, Rodos, Grčka, 16–20. 9. 2005.

Osnovni stručni termini:

- forge = kovati,
- quench = gasiti (vatru).

Main topics — Overview

- Introduction — industrial (or practical) background of the problem of "Cooling in Gas–Quenching Systems".
- Formulation of the problem in terms of a temperature distribution model in 3D.
- Model reduction from "intractable" 3D geometries to standard geometries in 1D.
- Numerical solution of 1D problems and implementation of some steps.
- Typical example and numerical results.
- Conclusion with some comments on applicability.

Introduction — Heat treatment

Heat treatment of metal parts is used to obtain required properties of treated materials. Primary target is to

- increase hardness of steel alloys.

This process is traditionally known as forging.

The final part of the treatment is quenching, or rapid cooling — traditionally, done in water or oil.

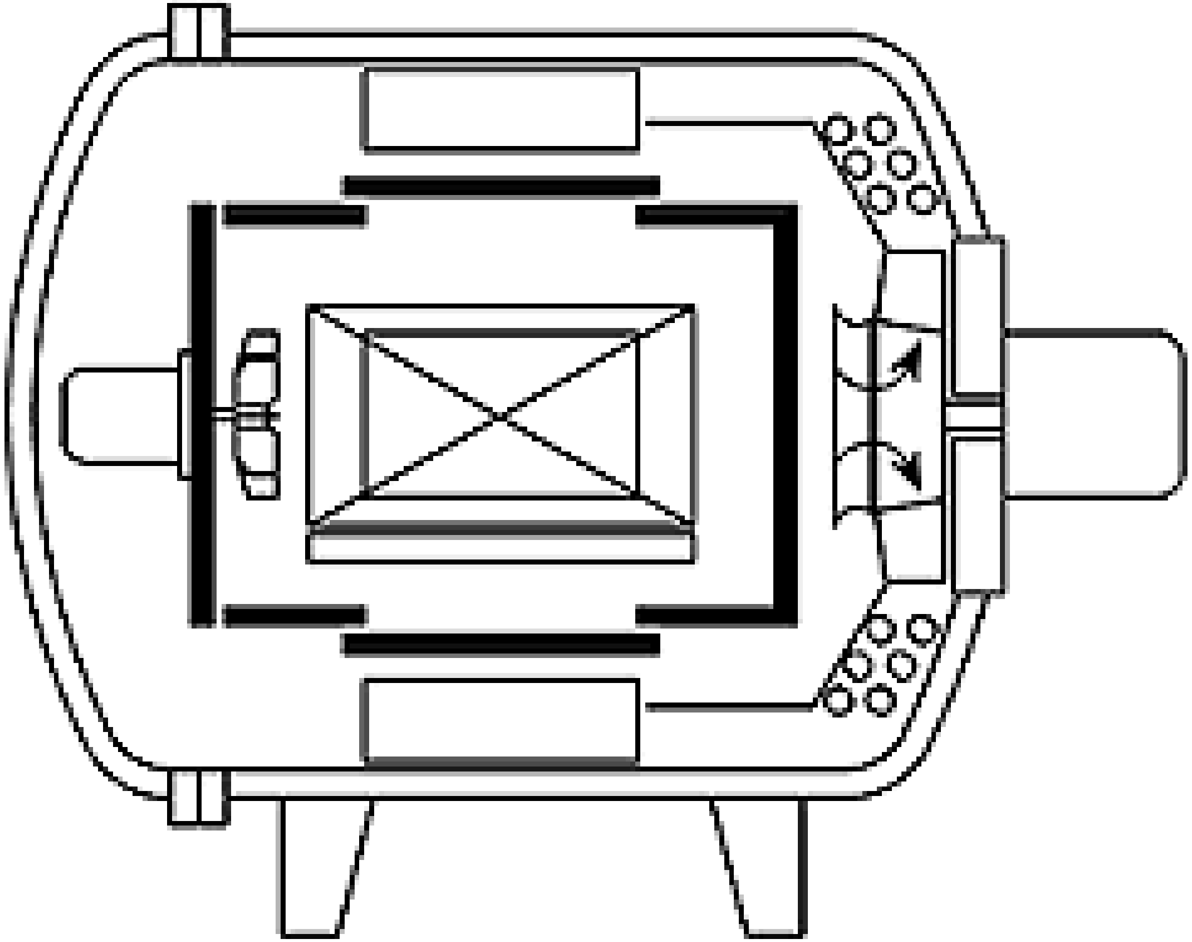
Modern technology also uses

- vacuum furnaces with high–pressure gas quenching

to cover a wide variety of heat treatment processes.

Introduction — Furnace

A typical furnace cross–section is:



Introduction — Examples (I)

Typical examples of heat treated materials are:

- aircraft engine parts,
- car parts (transmission, gear box),
- high quality tools.

Some of them are illustrated on the next few slides.

Introduction — Examples (II)

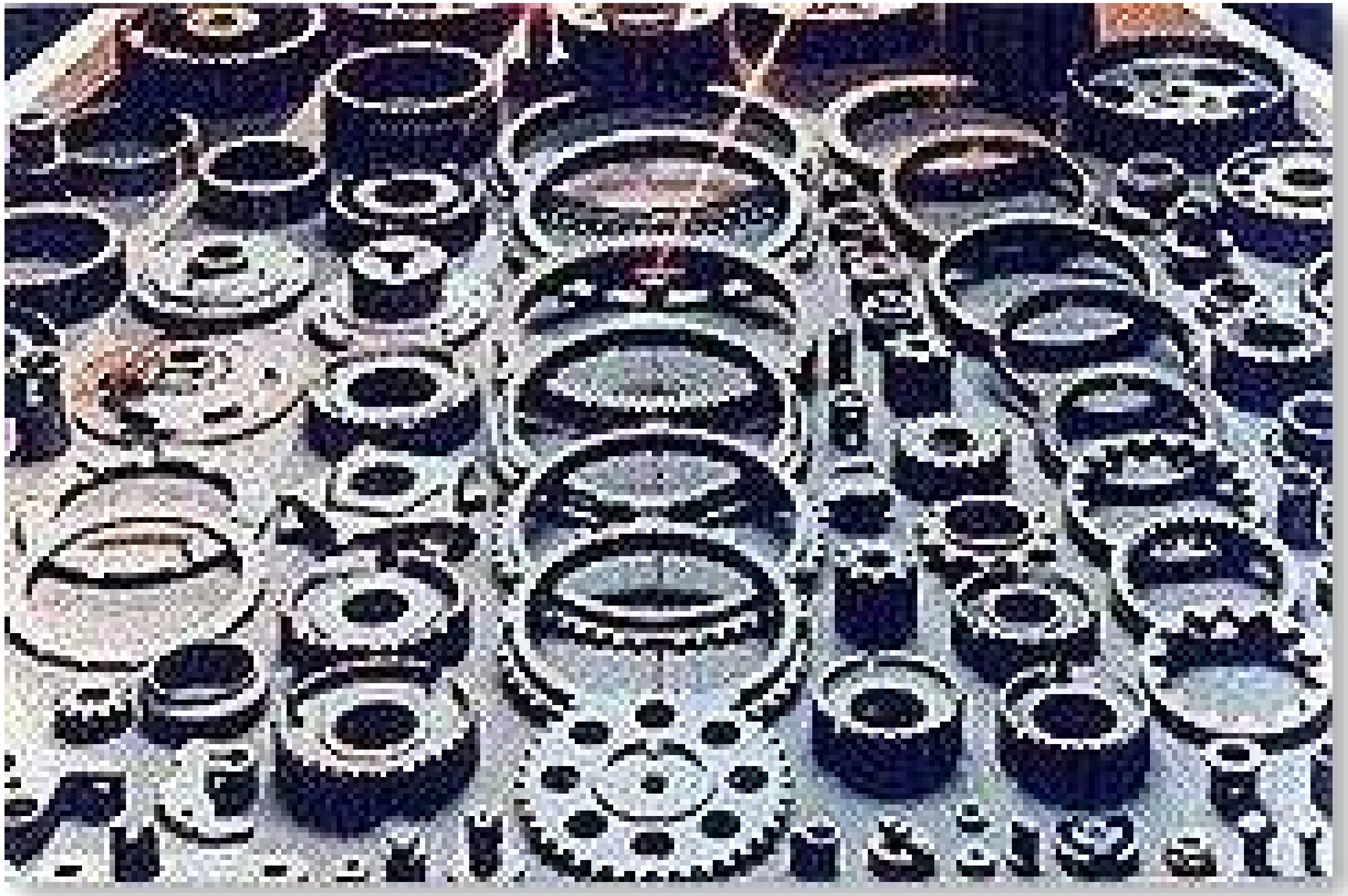
Vacuum–brazed turbine blades (in front of the furnace):

Introduction — Examples (III)

Plasma–carburized synchronizing rings:

Introduction — Examples (IV)

Vacuum–hardened gear parts:



Introduction — Problem and motivation

Rapid quenching can cause distortion of parts and development of residual stresses.

- In extreme cases, parts may crack during quenching!

Current state of heat treatment is still very much based on

- practical experience and technical know–how,

despite recent advances in technology.

Our immediate goal in this work is to develop techniques to

- predict the temperatures (and stresses)

in parts being heat treated and quenched.

Introduction — Goal

In the long run, this will be used in a "quenching expert system" which will be able to:

- predict the quality of the treated parts,
- plan quenching conditions for future treatments,
- optimize time, energy, cost — increase productivity.

The first steps in this process, as usual, are:

- selection of a tractable physical and mathematical model,
- extensive and accurate data gathering,
- model verification.

Temperature distribution model (I)

A part being quenched is represented by a domain $\Omega \subset \mathbb{R}^3$.

Temperature distribution in this part is determined by the heat conduction equation (HCE)

$$\rho c \frac{\partial T}{\partial t} = \operatorname{div}(\lambda \operatorname{grad} T),$$

with the following notation:

$x \in \Omega$ — space coordinates, each in [m],

$t \geq 0$ — time [s],

$T = T(x, t)$ — temperature [°C] or [K].

Temperature distribution model (II)

Physical properties of the part are:

- ρ — density $[\mathrm{kg/m^3}]$,
- c — specific heat $[\mathrm{J/(kg\,K)}]$,
- λ — thermal conductivity $[\mathrm{W/(m\,K)}]$.

The initial condition is the uniform temperature distribution

$$T(x,0) = T_0, \quad x \in \Omega.$$

(Realistic assumption — after the initial heat treatment, just before quenching, which starts at $t = 0$).

Typical values of T_0 can be as high as $1100\,^\circ\mathrm{C}$.

Temperature distribution model (III)

Note: Because of the temperature range involved, all physical properties

$$\rho, \quad c, \quad \lambda$$

are temperature dependent, and cannot be treated as constant. The whole problem is nonlinear!

Boundary conditions should represent the flow of thermal energy between

- the surface of the material ($\partial\Omega$) and
- the quenching medium — cooling gas, blown at high speed and pressure.

Boundary conditions (I)

Our model is the Newton's law of cooling on the boundary $\partial\Omega$

$$\lambda \frac{\partial T}{\partial n} = -\alpha \left(T - T_x\right),$$

where:

n — outer normal vector on $\partial\Omega$,

T_x — temperature of the cooling gas [°C] or [K],

α — Newton's boundary heat transfer coefficient [W/(m^2 K)].

Generally, both T_x and α depend on (x, t) on $\partial\Omega$.

Boundary conditions (II)

In this model of boundary conditions

- T_x can be regarded as known (say, measured),
- but α is unknown.

In other words, to use this model, we first have to find α, or

- solve the inverse problem!

One does not bother with solution of inverse problems, unless there is a very good motivation for doing so.

So, what do we expect to achieve by finding α?

Why α*?*

Our basic assumption is that

- α represents the overall quenching conditions quite well,

and can be used as a basis of an "expert quenching system".

We also expect that α (mildly) depends on

- material and geometry of various parts in the same load, and
- position of a particular part inside the chamber, due to somewhat different cooling conditions (gas flow).

Model verification

Of course, all these assumptions have to be verified in practice.

By gathering enough data for

- various loads,
- geometries and
- positions,

we may be able to automatically optimize the quenching conditions.

This outlines our global strategy towards the goal of an "expert quenching system".

Solution of the inverse problem

How to calculate α for a particular part, under particular cooling conditions?

The global numerical procedure is:

- measure temperatures near, or at the surface of the part,
- use these measurements as Dirichlet boundary conditions and solve the HCE,
- extend or extrapolate the solution towards the boundary (if necessary),
- calculate α via numerical differentiation.

Model reduction

Unfortunately, it is almost impossible to measure any boundary condition in a 3D space (intractable model)!

Therefore, we have to make one more simplification:

- reduce the intractable 3D model to one of the standard (and tractable) 1D models.

Two basic geometries which allow 1D models are:

- infinite plates,
- infinite cylinders.

We shall describe the α calculation procedure for "infinite" plates. The procedure for "infinite" cylinders is quite similar.

Infinite plate model (I)

In the infinite plate model we have:

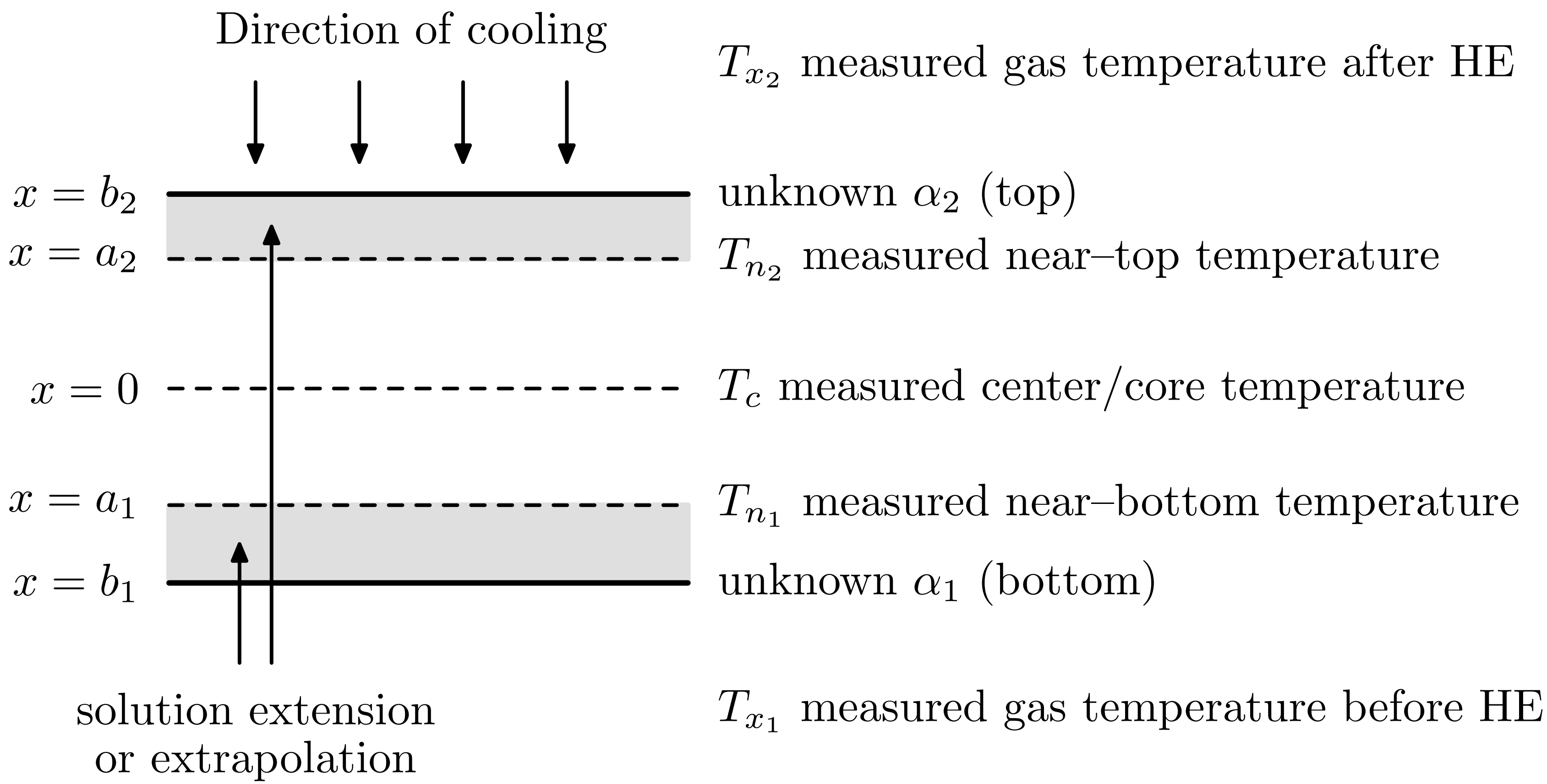
- a thin lying steel plate,
- initially heated to the uniform temperature T_0,
- cooled by gas blown from above the plate.

The gas then passes through a heat–exchanger (HE) and cools, before being blown again from the top.

The model situation is illustrated in the following figure, which shows

- the cross–section of the plate, and
- the positions of the temperature measurement devices.

Infinite plate model (II)



Measured temperatures (I)

The model itself requires only 4 measured temperatures

$$T_{n_1}, \quad T_{n_2}, \quad T_{x_1}, \quad T_{x_2}.$$

In practice, we also measure T_c at the core, which is taken as a reference, to check the results.

All temperatures are measured at discrete times,

- usually 1 s apart, but can be up to 10 s in some cases,
- until some final time t_{final}, typically 1800 s,
- and rounded to the nearest °C.

Measured temperatures (II)

Note that accurate measurement of surface temperatures (at b_1 and b_2) is virtually impossible.

The measurements are taken at points a_1 and a_2 beneath the surface (so $b_1 < a_1$, and $a_2 < b_2$).

The solution has to be extended from $[a_1, a_2]$ to $[b_1, b_2]$. This can be done by

- the quasi–reversibility method of Lattès and Lions, or
- simple extrapolation, if the depths $|b_i - a_i|$ are small, with respect to the whole thickness $b_2 - b_1$ of the plate.

Experiments show that simple extrapolation is sufficient for depths $\leq 10\%$ of the thickness. Works up to 20%.

Numerical solution (I)

The actual computation is performed in two phases.

Phase 1: α calculation (inverse problem), $t \in [0, t_{\text{final}}]$:

- solve the HCE on $[a_1, a_2]$ with measured temperatures T_{n_1} and T_{n_2} as boundary conditions,
- extend or extrapolate the solution to $[b_1, b_2]$,
- calculate α_1, α_2, using measured cooling gas temperatures T_{x_1} and T_{x_2}, respectively,
- check errors in calculated core temperature with respect to T_c (reference point).

Numerical solution (II)

Phase 2: Model verification (direct problem), $t \in [0, t_{\text{final}}]$:

- solve the HCE on $[b_1, b_2]$ with the law of cooling boundary conditions, using calculated α_1, α_2 (from Phase 1),
- check errors in calculated temperatures at a_1, core and a_2, with respect to T_{n_1}, T_c and T_{n_2} (reference points).

Numerical solution (III)

Accurate and efficient implementation of both phases involves several interesting numerical problems.

- All measured temperatures (except T_c) have to be smoothed before use. This is done by
  - cubic spline least–squares approximation (Dierckx).
- The same applies to all calculated $\alpha(t)$ values.
- The nonlinear implicit method is used to solve the HCE (in both phases), with simple iterations per time–step.
- Numerical differentiation (needed for α) is based on low–degree polynomial least–squares approximation (not interpolation).

Numerical example — Data (I)

A number of experiments has been carried out with thin plates and thin cylinders.

As an example, we take a lying steel alloy plate.

- Dimensions of the plate are: $20\,\mathrm{cm} \times 20\,\mathrm{cm} \times 5\,\mathrm{cm}$, or

$$b_1 = -0.025\,\mathrm{m}, \quad b_2 = 0.025\,\mathrm{m}.$$

- Near–surface temperatures T_{n_1} and T_{n_2} are measured at the depth of $4.5\,\mathrm{mm}$, so

$$a_1 = -0.0205\,\mathrm{m}, \quad a_2 = 0.0205\,\mathrm{m}.$$

Numerical example — Data (II)

- All measured temperatures have 184 data points, with $\Delta t \approx 10\,\mathrm{s}$.
- Final time: $t_{\mathrm{final}} = 1871\,\mathrm{s}$.
- Initial condition: $T_0 = 1099\,^\circ\mathrm{C}$.
- Cooling conditions: Nitrogen N_2, blown from above,
  - at fan speed of 3000 rotations per minute,
  - with varying pressure:

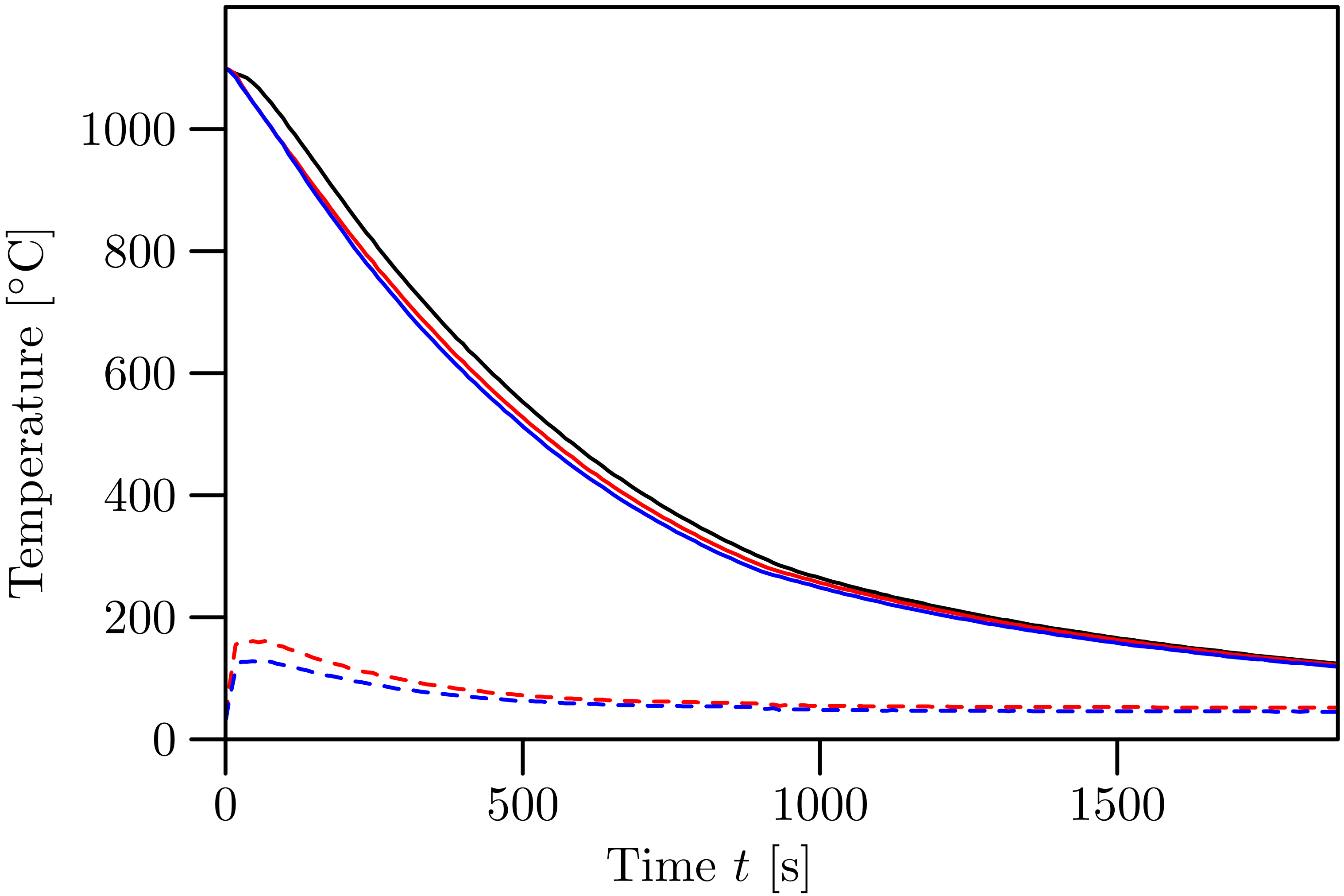
$$6.0\,\text{bar for the first } 900\,\text{s},$$

$$3.2\,\text{bar later on.}$$

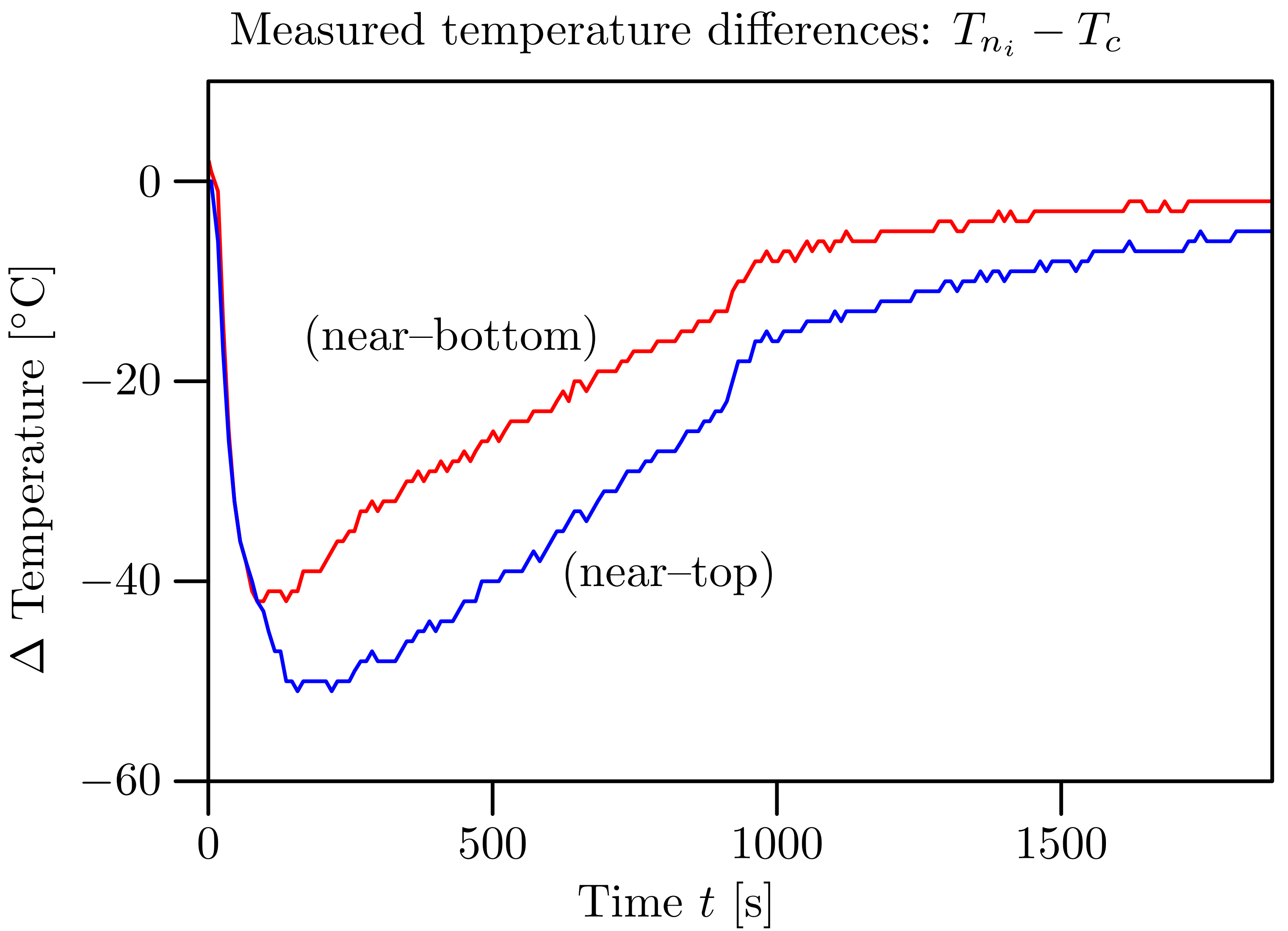
This change in pressure should be reflected in α.

Numerical example — Data (III)

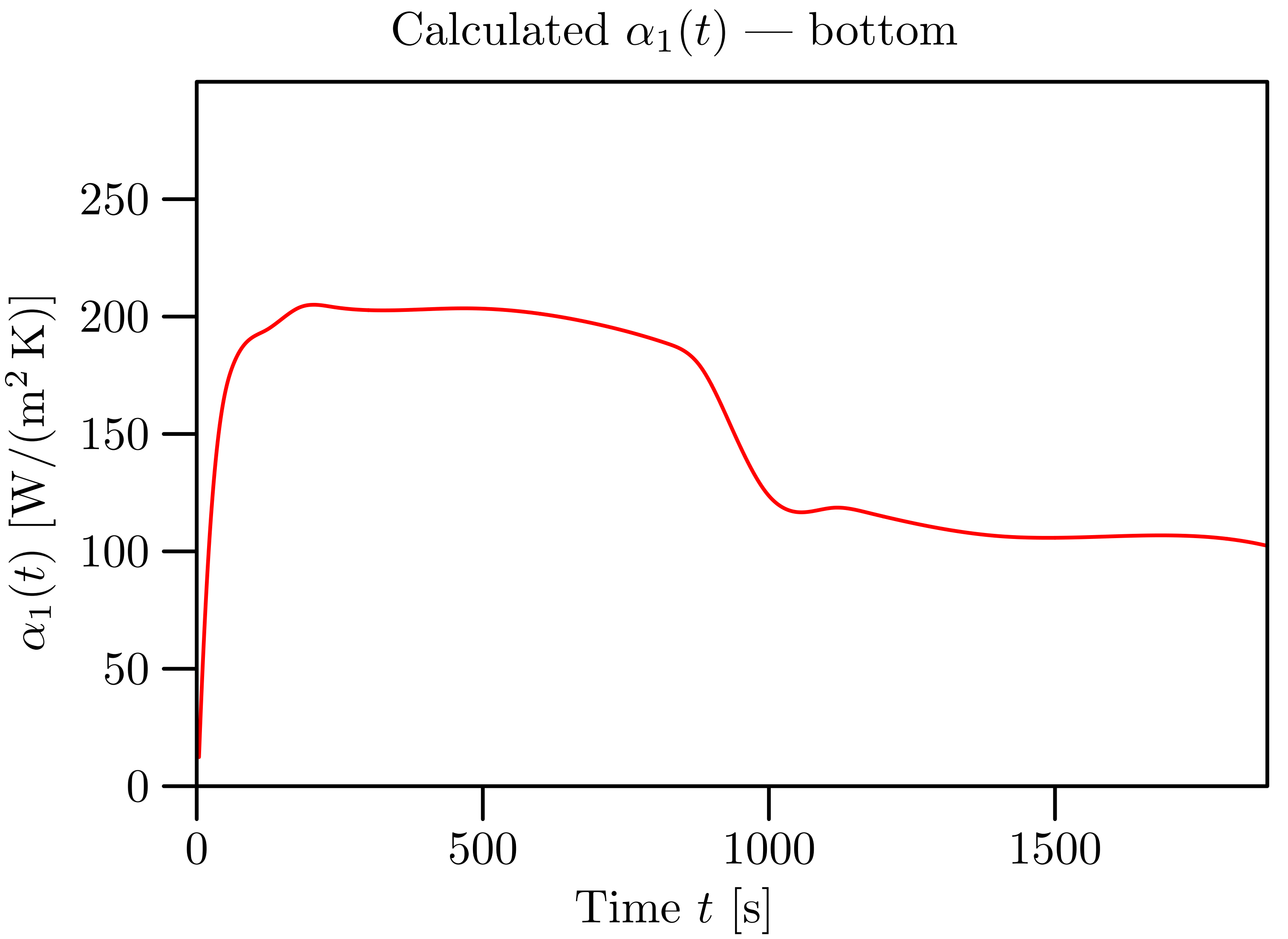
Measured temperatures: T_c (core), T_{n_1} (near–bottom), T_{n_2} (near–top)



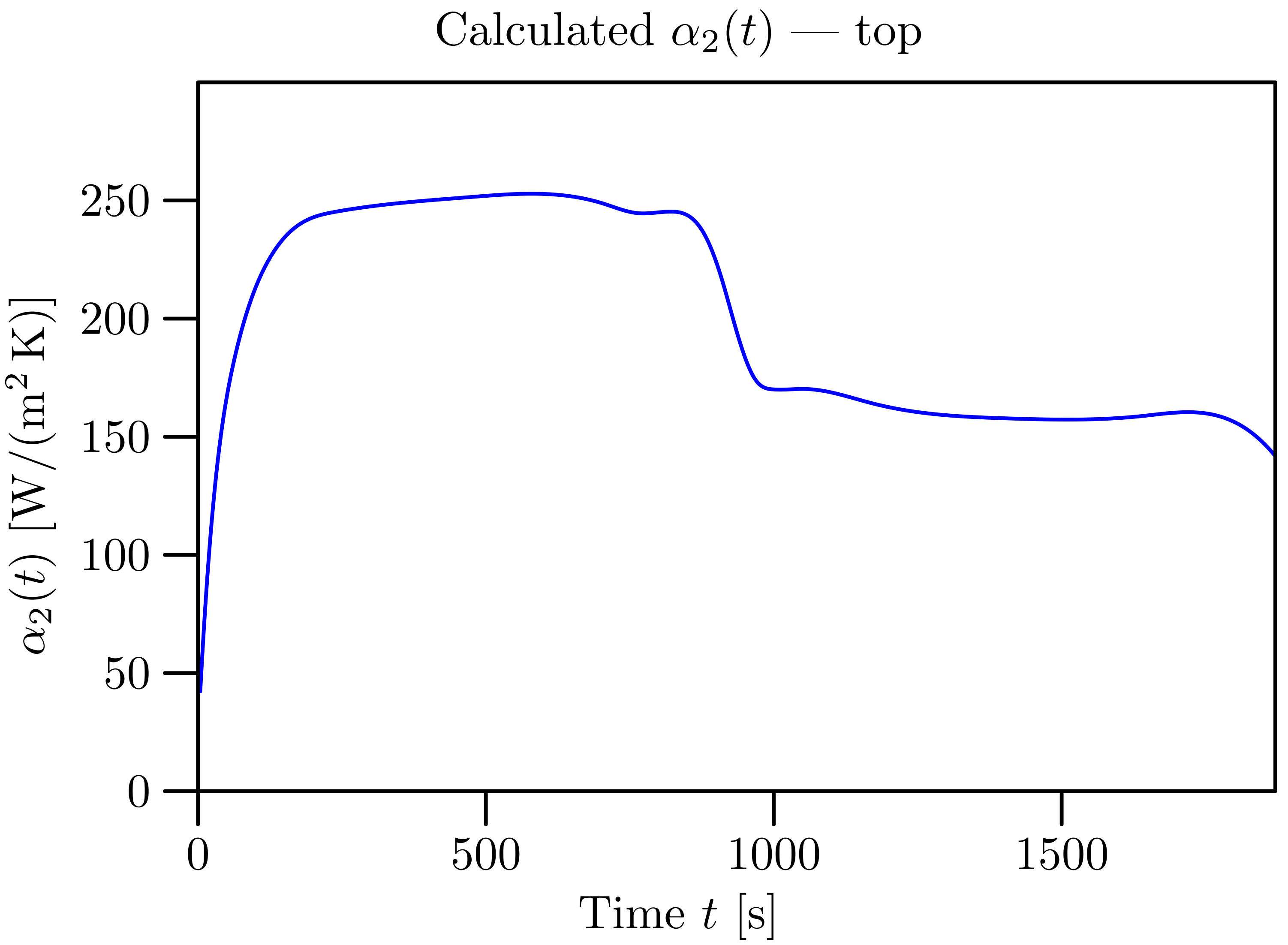
Numerical example — Data (IV)



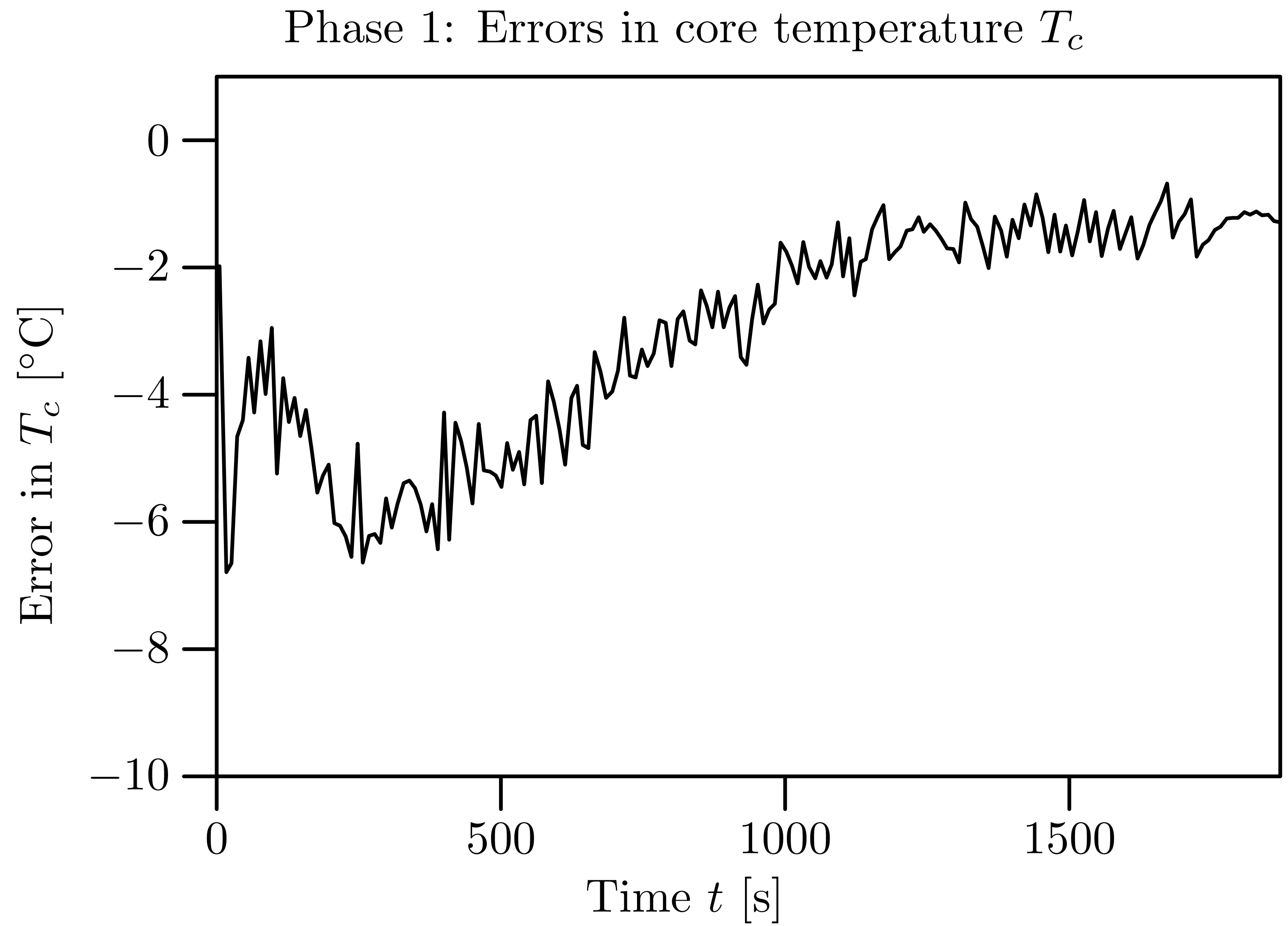
Numerical example — Results (I)



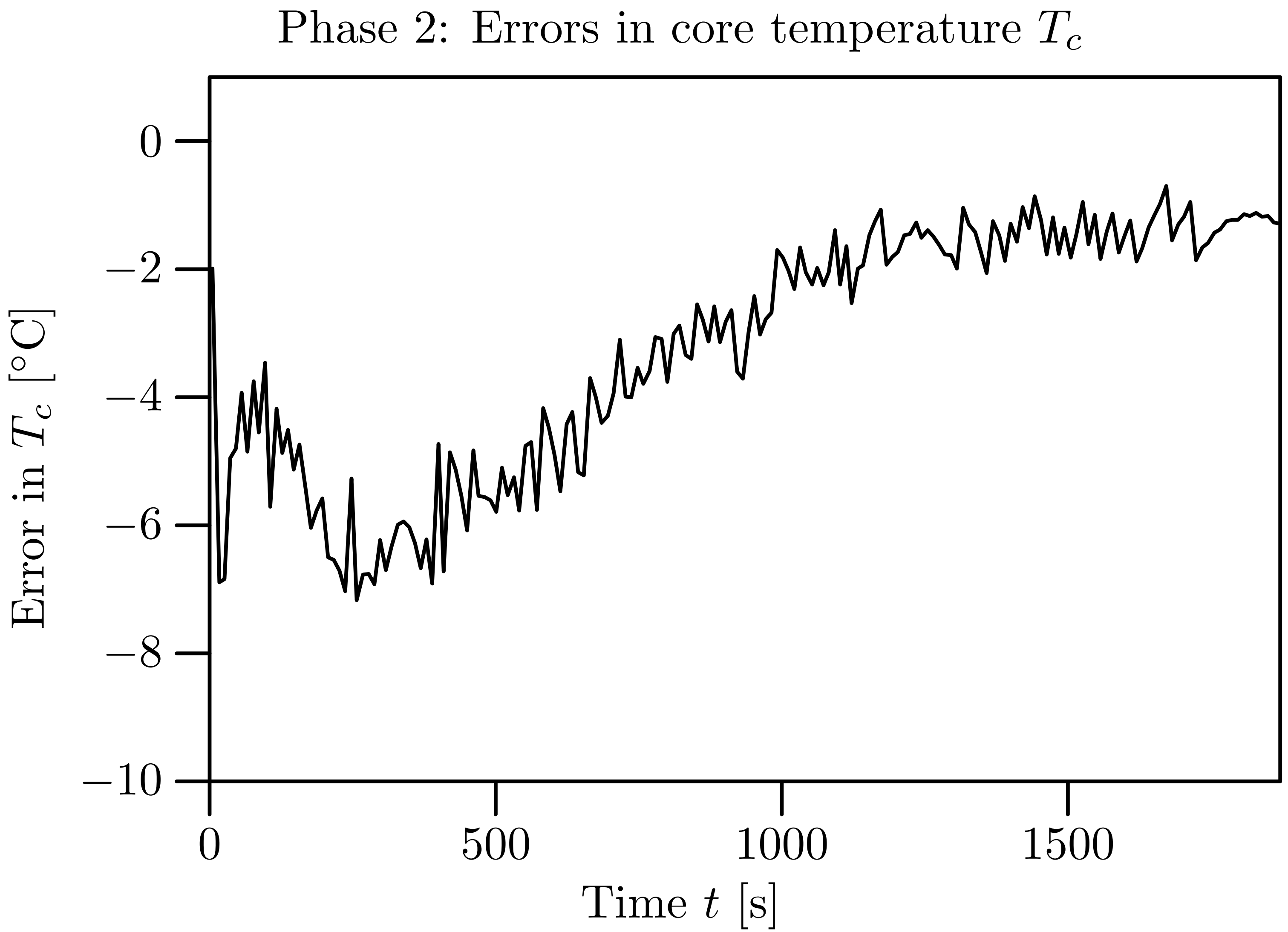
Numerical example — Results (II)



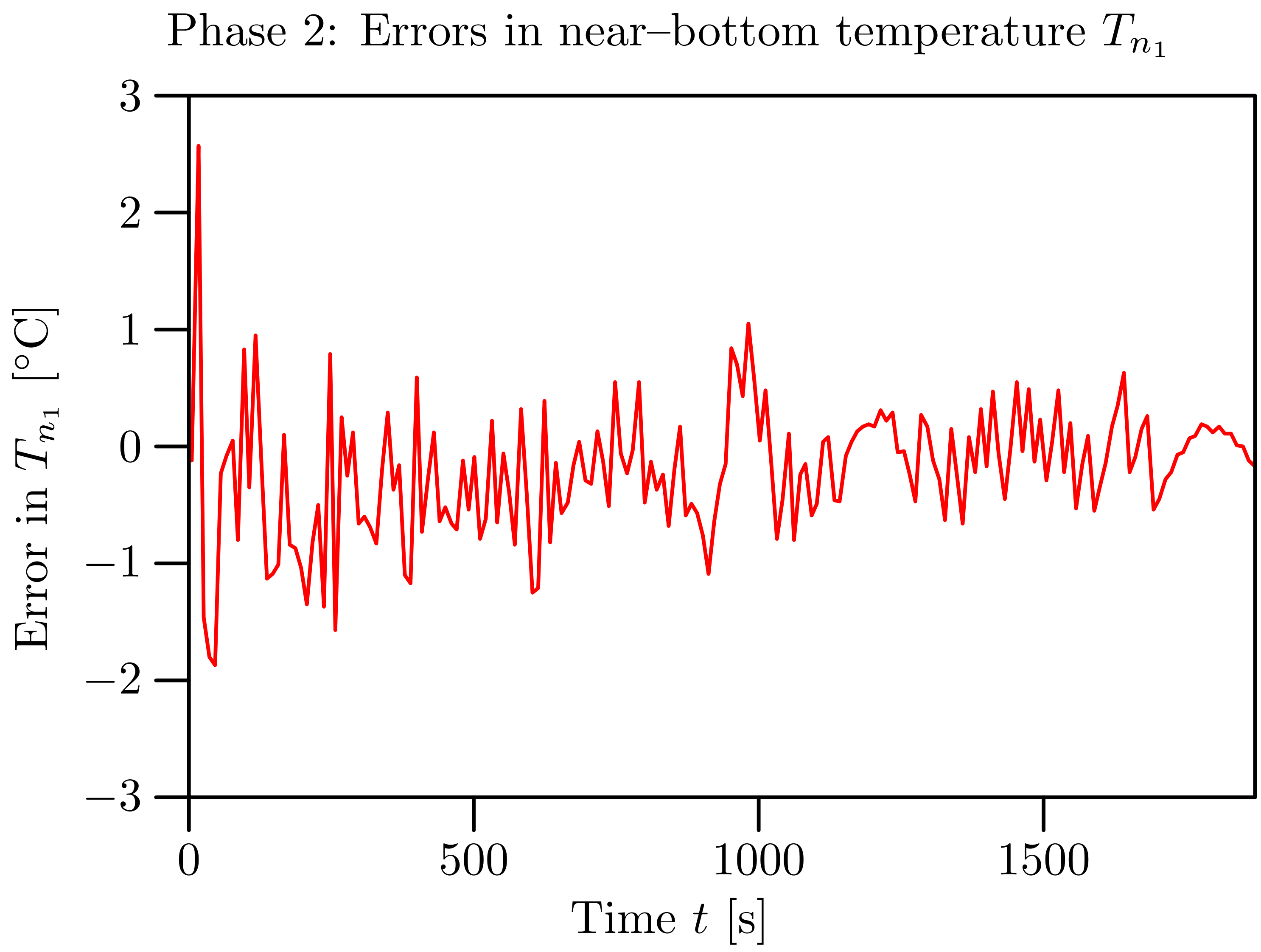
Numerical example — Results (III)



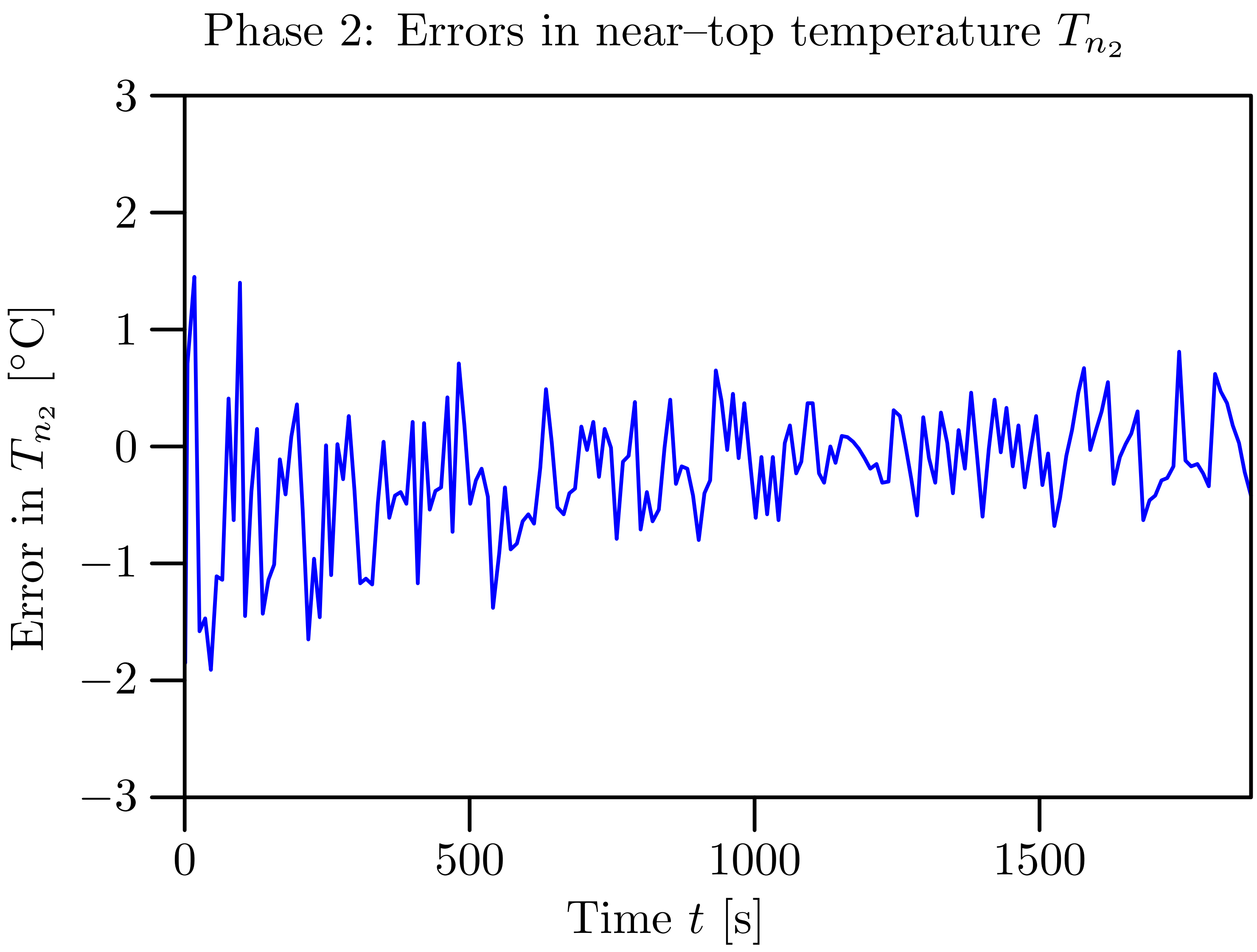
Numerical example — Results (IV)



Numerical example — Results (V)



Numerical example — Results (VI)



Comments on the results (I)

These comments apply generally to all experiments that have been carried out.

Calculated errors refer to original measured temperatures, so they include:

- smoothing of measured temperatures,
- numerical solution of the nonlinear HCE (with simple iterations),
- smoothing of calculated α values.

Comments on the results (II)

Calculated errors in the core temperature T_c are almost equal for both phases. In other words,

- the change of boundary conditions between two phases and α smoothing, together, introduce very small (negligible) errors.

However, the errors themselves are not negligible.

- We have an over–estimate of T_c — the core cools more quickly than predicted (which is good).
- This is probably due to inaccurate physical properties at high temperatures.

Comments on the results (III)

We had to use very crude approximations, because

- accurate data (especially for c) are very hard to get.

This is a problem for national standard institutes!

In Phase 2, calculated errors in near–surface temperatures T_{n_1} and T_{n_2} are mostly initial smoothing errors, so

- α calculation (in Phase 1) is quite accurate and smoothing errors in α are really negligible.

These results confirm that calculated α curves can be used to predict temperatures, or

1D model works on “near 1D” geometries.

Conclusion (I)

For practical applications, this 1D model has many deficiencies.

On the other hand, 3D (and even 2D) models are out of question for everyday use, due to:

- lack of data,
- computational cost (time).

To minimize some of 1D model deficiencies:

- a cylindrical "flux–sensor" has been built for data gathering in all production loads (charges).

Conclusion (II)

The "flux–sensor":

Conclusion (III)

Some advatanges of the “flux–sensor”:

- a few of them can be placed at different positions inside the chamber,
- calculated “benchmark” α curves for each load are stored in a data base which is used for quenching optimization.

Acknowledgements (I)

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