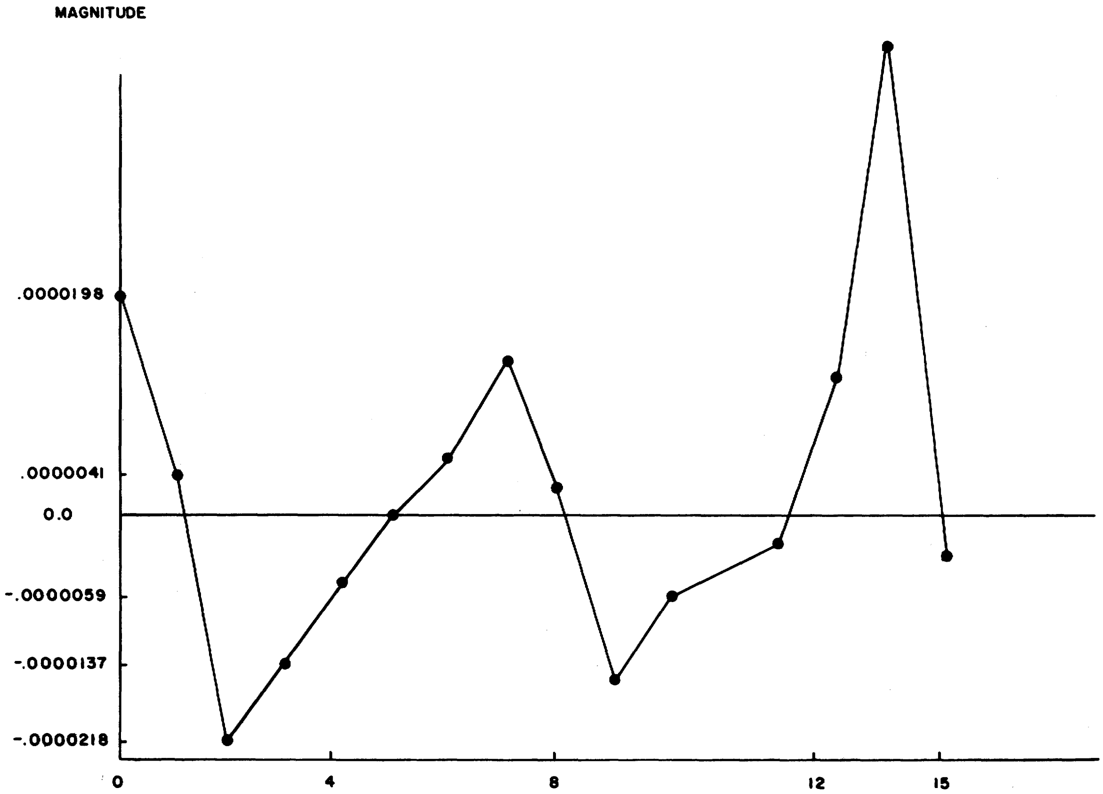

Fig. 11. Result of filtering in the Walsh domain.

Future work in this area will include the development of user oriented program packages to make conversion from the Fourier and the other unitary transforms to the walsh domain.

REFERENCES

[1] A. E. Kahveci and E. L. Hall, "Frequency domain design of sequency filters," presented at the 1972 Walsh Function Symp., Washington, D. C.
[2] W. K. Pratt, "Generalized Wiener filtering computation techniques," in *Proc. UMC Two-Dimensional Digital Signal Processing Conf.*, Oct. 1971, pp. 1-1-1, pp. 1-1-10.
[3] B. Gold and C. M. Radar, *Digital Signal Processing.* New York: McGraw-Hill, 1969, pp. 173–181.
[4] H. C. Andrews, *Computer Techniques in Image Processing.* New York: Academic, 1970, pp. 105–131.
[5] J. L. Walsh, "Remarks on the history of Walsh functions," in *Proc. Walsh Function Symp.*, Washington, D. C., Apr. 1970.
[6] W. K. Pratt, "Linear and nonlinear filtering in the Walsh domain," presented at the 1971 Symp. Applications of Walsh Functions, Washington, D. C.

Divide-and-Correct Algorithm for Division in a Negative Base

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***Abstract*—A divide-and-correct algorithm is described for multiple-precision division in the negative base number system. In this algorithm an initial quotient estimate is obtained from suitable segmented operands; this is then corrected by simple rules to arrive at the true quotient.**

***Index Terms*—Divide-and-correct algorithm, negative base, polarization, segmented operands, sign correction.**

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I. INTRODUCTION

Recently, a class of divide-and-correct algorithms have been described for division in conventional and signed digit number systems [1], [2]. In this correspondence we describe an algorithm for division in the negative base number system [3].

II. NOTATIONS AND DEFINITIONS

1) *Representation:* Let A, B, and Q, respectively, be dividend, divisor, and quotient [in floating point form with integral mantissa in radix $(-\beta)$] given by:

$$A = (-\beta)^{e_a}\cdot a = (-\beta)^{e_a}\sum_{j=0}^{n} a_j(-\beta)^j \tag{1a}$$

$$B = (-\beta)^{e_b}\cdot b = (-\beta)^{e_b}\sum_{j=0}^{d} b_j(-\beta)^j \tag{1b}$$

$$Q = (-\beta)^{e_q}\cdot q = (-\beta)^{e_q}\sum_{j=0}^{m} q_j(-\beta)^j. \tag{1c}$$

2) *Polarization Overflow:* Polarization is an operation which reverses the sign of a number in the negative base [3]. The polarized form of "a" is denoted by $\bar{a}$. When an m-precision number is polarized it either contracts to an $(m-1)$ precision number or expands to an $(m+1)$ precision number; in the latter case, the leading digit will be unity and this is defined as "polarization overflow."

3) *Rounding-Off Operation:* The rule for rounding an $(m + 1)$ precision number

$$Z = (-\beta)^{e_z} \sum_{j=0}^{m} z_j(-\beta)^j$$

into an $(n + 1)$ precision number Z_R is as follows.

a) If $z_{m-n-1} \geq \beta/2 + 1$ (β even) or $(\beta + 1)/2$ (β odd) then subtract unity from z_{m-n} and truncate the $(m - n)$ precision tail part.

b) Otherwise, i.e., if $z_{m-n-1} \leq \beta/2$ (β even) or $(\beta - 1)/2$ (β odd), then truncate the $(m - n)$ precision tail part. The limits of roundoff error $(Z_R - Z)$ in the nth position is given by

$$\left.\begin{array}{l} -\dfrac{1}{2} + \dfrac{1}{\beta(\beta+1)} \leq (Z_R - Z)(-\beta)^{-(m-n)} \leq \dfrac{1}{2} + \dfrac{1}{\beta(\beta+1)} \quad (\beta \text{ even}) \\ \text{and} \\ -\dfrac{1}{2} - \dfrac{\beta-1}{2\beta(\beta+1)} \leq (Z_R - Z)(-\beta)^{-(m-n)} \leq \dfrac{1}{2} - \dfrac{\beta-1}{2\beta(\beta+1)} \quad (\beta \text{ odd}). \end{array}\right\} \quad (2)$$

III. THE SEGMENTED DIVISION ALGORITHM AND LOGIC OF QUOTIENT SELECTION

The $(d + 1)$ precision mantissa b of the divisor is rounded into a single precision number b_R. The leading two digits (or three, in the case of a polarization overflow) of the partial remainder R_j ($R_n = a$, the mantissa of the dividend) are taken and fully divided by b_R to obtain a single precision positive or double precision (arising due to a polarized form) negative quotient q_j and remainder r_j. The partial remainder R_{j-1} is computed using the recursion

$$R_{j-1} = R_j - q_j b(-\beta)^{j-d} \qquad (j = n, n-1, \cdots, n-m). \quad (3)$$

Proceeding exactly in the same manner, as in [2], we get

$$R_{j-1} = [q_j(b_R - b) + r_j](-\beta)^{j-1} + a_{j-2}(-\beta)^{j-2} + \cdots + a_0. \quad (4)$$

Our assumption that the magnitude of each quoteint digit is $\leq (\beta - 1)$ is satisfied if 1) at the start we restrict

$$q_n \leq (\beta - 1) \quad (5)$$

by choosing one auxiliary leading zero digit at the left of the dividend and 2) the leading two (or three, in case of polarization overflow) digits of every R_{j-1} denoted by $[R_{j-1}]_T$ obey the inequality

$$[R_{j-1}]_T = |[q_j(b_R - b) + r_j](-\beta) + a_{j-2}| \leq (\beta - 1)b_R + r_{j-1}. \quad (6)$$

Note that from (2)

$$\left.\begin{array}{l} -\dfrac{1}{2} + \dfrac{1}{\beta(\beta+1)} \leq (b_R - b) \leq \dfrac{1}{2} + \dfrac{1}{\beta(\beta+1)} \quad (\beta \text{ even}) \\ \text{and} \\ -\dfrac{1}{2} - \dfrac{\beta-1}{2\beta(\beta+1)} \leq (b_R - b) \leq \dfrac{1}{2} - \dfrac{\beta-1}{2\beta(\beta+1)} \quad (\beta \text{ odd}) \end{array}\right\} \quad (7)$$

for rounding off the divisor, provided e_b is replaced by $e_b + d$.

The inequality (6) can be satisfied by choosing a q_j such that, if $q_j \neq 0$ and $r_j \neq 0$, then

$$\operatorname{sgn} q_j \cdot \delta \neq \operatorname{sgn} r_j \quad (8)$$

where $\delta = (b_R - b)$.

This is defined as a "sign-dictation" scheme [2]. Note that δ is positive if $b_{d-1} \leq \beta/2$ (β even) or $(\beta - 1)/2$ (β odd) and negative if $b_{d-1} \geq \beta/2 + 1$ (β even) or $(\beta + 1)/2$ (β odd). Also

$$|r_j| \leq (b_R - 1). \quad (9)$$

The condition under which (6) is satisfied subject to sign-dictation scheme is given by:

$$\begin{array}{l} \text{if } \delta \text{ is positive } (\geq 0)\text{, then } b_d \geq \dfrac{\beta}{2} + 1 \ (\beta \text{ even}) \text{ or } \dfrac{\beta+1}{2} \ (\beta \text{ odd}); \\ \text{if } \delta \text{ is negative then } b_d \geq \dfrac{\beta}{2} + 2 \ (\beta \text{ even}) \text{ or } \dfrac{\beta+3}{2} \ (\beta \text{ odd}). \end{array} \quad (10)$$

In order to carry out the sign-dictation scheme, it is convenient to use the following rule.

If r_j is not of the required sign it can be modified by adding a sign correction to q_j, as specified in Table I (for $q_j = 0$ or $r_j = 0$, this correction is not required).

IV. ALLOWED DIVISOR PATTERNS AND CHOICE OF MULTIPLIER FOR OBTAINING THE ALLOWED PATTERN

Table II lists all the allowed leading two (or three if $b_d = 1$) digit patterns of "b" which satisfy (10). It is to be noted that the polarized forms of divisors listed in Table II also satisfy (10. In case the given divisor or its polarized form do not satisfy the requirement, we premultiply b by a certain multiplier α so that the product has the allowed pattern of Table II. The dividend is also then premultiplied by α to get the correct quotient. A suitable choice of α for possible divisors b in a general base $-\beta$ is presented in Table III.

V. CONVERSION OF SIGNED DIGIT QUOTIENTS TO THE CONVENTIONAL FORM

Since positive (single digit) and negative (two digit) q_j's arise, it is necessary to convert these to the conventional form in the negative base. This is done at the termination of the division operation by polarizing and adding the negative quotients at the appropriate digit positions.

VI. ALGORITHM AND EXAMPLE

Step 1: Check whether the leading two (or three in case $b_d = 1$) digits of the divisor are among the patterns listed in Table II. If so, execute Step 2, otherwise choose a suitable multiplier, as specified by Table III to transform the divisor into the required range given in Table II. Multiply both the divisor and the dividend by this multiplier and take the multiplied divisor and the dividend as "b" and "a," respectively.

Step 2: Add one leading zero digit to the dividend (initial partial remainder R_j, $j = n$) for overflow protection.

Step 3: Roundoff the leading two characters of the divisor into a single precision number b_R and determine the sign of δ $(= b_R - b)$.

Step 4: Take the leading two digits (three in the case of polarization overflow) of the partial remainder R_j (the dividend itself for $j = n$) and divide by b_R. Let q_j be the quotient and r_j be the remainder of this division. If $r_j \neq 0$ and $q_j \neq 0$ then check whether the sign of r_j and the sign of $q_j(b_R - b)$ are opposite. If so, execute Step 5; otherwise apply the sign correction to q_j as specified by Table I, and go to Step 5.

Step 5: Compute

$$R_{j-1} = R_j - q_j b(-\beta)^{j-d}$$

and go to Step 4, and reiterate.

Step 6: If the division is stopped at the $(n - m)$th step after obtaining q_{n-m}, then

$$e_q = (e_a - e_b) + n - m - (d + 1)$$

where $(n + 1)$, $(d + 1)$, and $(m + 1)$ denote the number of digits in the dividend (including the auxiliary zero), divisor, and quotient, respectively.

Example: The numbers are in base -10. $a = 188692$; $e_a = 0$; $b = 48$; $e_b = 0$; $\alpha = 2$, $b \cdot \alpha = 76$, $b_R = 6$, $a \cdot \alpha = 155184$. Sign of $(b_R - b)$ is negative.

$$n - m = 2; \quad (d + 1) = 2; \quad q = 0094(19) = 959; \quad e_q = 0.$$

TABLE I

Sign of $q_j \cdot \delta$	Sign of r_j	Sign Correction
0	1	0
1	0	0
0	0	+1
1	1	−1

0 = positive, 1 = negative.

TABLE II
$0 \leq \gamma \leq (\beta - 1)$

β	b_d	b_{d-1}	b_{d-2}
Even	1	1	$\geq (\beta/2) + 1$
	1	0	$\leq (\beta/2)$
	$(\beta/2) + 1$	$\leq (\beta/2)$	γ
	$\geq (\beta/2) + 2$	γ	γ
Odd	1	1	$\geq (\beta + 1)/2$
	1	0	$\leq (\beta - 1)/2$
	$(\beta + 1)/2$	$\leq (\beta - 1)/2$	γ
	$\geq (\beta + 3)/2$	γ	γ

TABLE III
$0 \leq \gamma \leq (\beta - 1)$

β	b_d	b_{d-1}	b_{d-2}	α
Even	1	1	$\leq \beta/2$	$(\beta/2) + 2$
	1	0	$\geq(\beta/2) + 1)$	$[(\beta/2) + 1](-\beta) + (\beta/2)$
	2	$\geq(\beta/2) + 1$	γ	$(\beta/2) + 1$
	2	$\leq(\beta/2)$	γ	$\beta/2$
	$3 \leq b_d \leq(\beta/2)$	γ	γ	$\left\lceil \frac{\beta^3 + 2\beta^2 + \beta - 2}{2\beta[(\beta + 1)b_d - \beta]} \right\rceil < \alpha \leq \left\lceil \frac{2\beta^3 + 3\beta^2 + \beta - 2}{2\beta[(\beta + 1)b_d + 1]} \right\rceil$
	$(\beta/2) + 1$	$\geq(\beta/2) + 1$	γ	$2(-\beta) + (\beta/2)$
Odd	1	1	$\leq(\beta - 1)/2$	$(\beta + 3)/2$
	1	0	$\geq(\beta + 1)/2$	$[(\beta + 1)/2](-\beta) + [(\beta - 1)/2]$
	2	$\geq(\beta + 1)/2$	γ	$(\beta + 1)/2$
	2	$\leq(\beta - 1)/2$	γ	$(\beta - 1)/2$
	$3 \leq b_d \leq(\beta - 1)/2$	γ	γ	$\left\lceil \frac{\beta^3 + \beta^2 + \beta + 1}{2\beta[(\beta + 1)b_d - \beta]} \right\rceil < \alpha \leq \left\lceil \frac{2\beta^3 + 3\beta^2 + 2\beta - 1}{2\beta[(\beta + 1)b_d + 1]} \right\rceil$
	$(\beta + 1)/2$	$\geq(\beta + 1)/2$	γ	$2(-\beta) + [(\beta - 1)/2]$

$\lceil x \rceil$ denotes greatest integer not greater than x.

j	6	5	4	3	2	1	0	q_j
R_6	0	**1**	**5**	5	1	8	4	0
$q_6 \cdot b$	0	0	0	0	0	0	0	
R_5		**1**	**5**	5	1	8	4	0
$q_5 \cdot b$		0	0	0	0	0	0	
R_4		**1**	**5**	**5**	1	8	4	9
$q_4 \cdot b$		0	6	3	6	0	0	
R_3			**1**	**8**	**7**	8	4	4
$q_3 \cdot b$			0	3	5	6	0	
R_2					**1**	**4**	4	19
$q_2 \cdot b$						7	6	
						0	0	

Note: The leading two (or three) digits taken for division by b_R are **underlined.**

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REFERENCES

[1] E. V. Krishnamurthy, "On a divide and correct method for variable precision division," *Commun. Assoc. Comput. Mach.*, vol. 8, pp. 179–181, Mar. 1965.
[2] ——, "Carry–borrow free sequential quotient generation with segmented signed digit operands," *Int. J. Contr.*, vol. 12, pp. 81–93, 1971.
[3] P. V. Sankar, S. Chakrabarti, and E. V. Krishnamurthy, "Arithmetic algorithms in a negative base," *IEEE Trans. Comput.*, vol. C-22, pp. 120–125, Feb. 1973.

A Numerical Algorithm for the Resolution of Boolean Equations

WALTER DEL PICCHIA

***Abstract*—This correspondence presents the remainder-quotient (RQ)-algorithm for the resolution of Boolean equations. If $w = w(x_{k_1}, x_{k_2}, \cdots x_{k_N}) = 1$ is the Boolean equation equivalent to**

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