Brza iterativna varijanta algoritma za DFT_n, $n=2^m$

U daljnjem stalno pretpostavljamo da je n potencija od 2, tj. $n=2^m$, $m\in\mathbb{N}$.

Da bismo dobili iterativnu varijantu algoritma iz rekurzivne, treba "raspakirati" rekurziju, tj. prvo pogledati što se događa na pojedinim nivoima rekurzije.

Rekurzivni algoritam ima tipični oblik obilaska binarnog stabla i to u "post"-uređaju – prvo obilazimo <u>oba</u> djeteta, obavljamo neki posao i vraćamo se natrag.

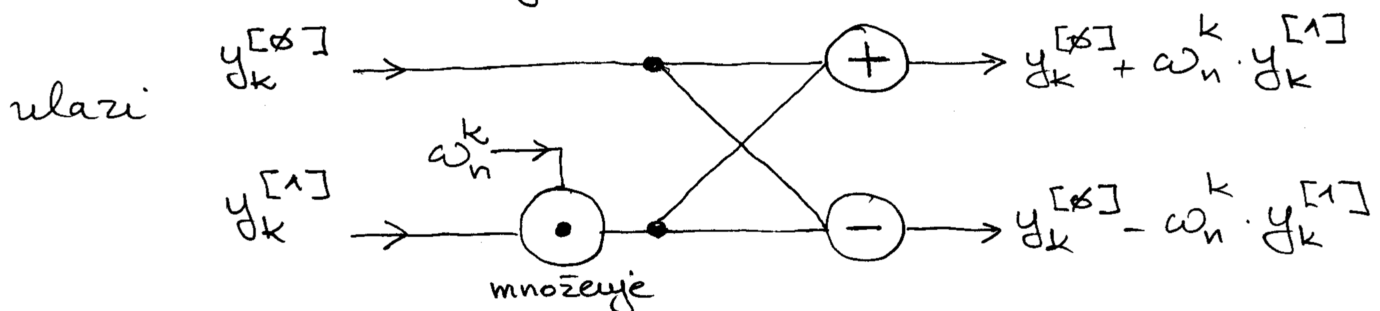
Naš "neki" posao je tzv. "butterfly" ili leptir operacija u unutarnjoj petlji, nakon rekurzivnih poziva.

Tu operaciju možemo shematski prikazati kao kombinatorni ili elektronički sklop (engl. "circuit) koji iz 2 ulaza $y_k^{[\emptyset]}$ i $y_k^{[1]}$, uz zadani faktor ω_n^k, generira 2 izlaza zadana (definirana) relacijama

$$y_k = y_k^{[\emptyset]} + \omega_n^k \cdot y_k^{[1]}$$

$$y_{k+n/2} = y_k^{[\emptyset]} - \omega_n^k \cdot y_k^{[1]} .$$

Standardna shema je:



Ove krugove možemo zamišljati i kao vrlo jednostavne procesore (krugove), koje možemo međusobno vezati da napravimo računalo ("veliki" krug) za računanje DFT_n.

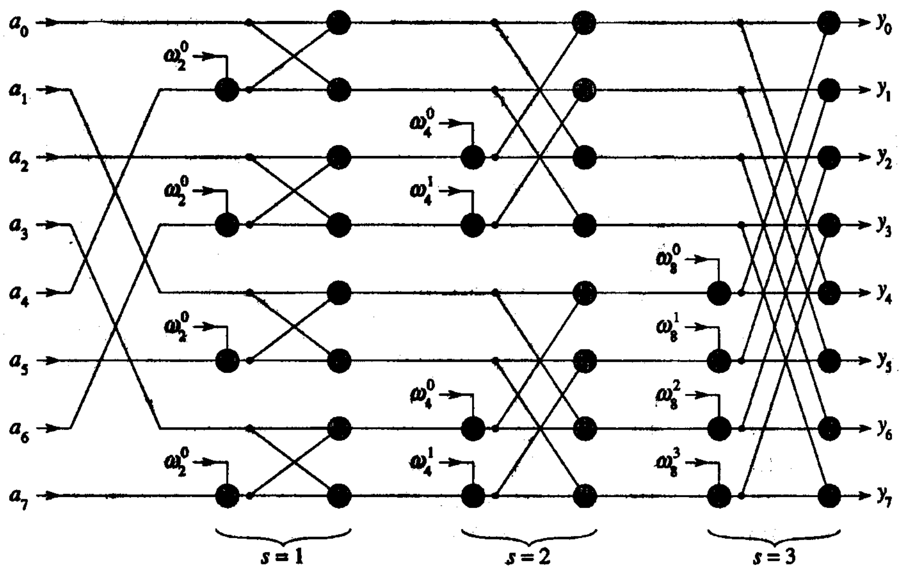


Figure 32.5 A combinational circuit PARALLEL-FFT that computes the FFT, here shown on $n = 8$ inputs. The stages of butterflies are labeled to correspond to iterations of the outermost loop of the FFT-BASE procedure. An FFT on n inputs can be computed in $\Theta(\lg n)$ depth with $\Theta(n \lg n)$ combinational elements.

formed in parallel. The value of s in each iteration within FFT-BASE corresponds to a stage of butterflies shown in Figure 32.5. Within stage s, for $s = 1, 2, \ldots, \lg n$, there are $n/2^s$ groups of butterflies (corresponding to each value of k in FFT-BASE), with 2^{s-1} butterflies per group (corresponding to each value of j in FFT-BASE). The butterflies shown in Figure 32.5 correspond to the butterfly operations of the innermost loop (lines 8–11 of FFT-BASE). Note also that the values of ω used in the butterflies correspond to those used in FFT-BASE: in stage s, we use $\omega_m^0, \omega_m^1, \ldots, \omega_m^{m/2-1}$, where $m = 2^s$.

Exercises

32.3-1
Show how ITERATIVE-FFT computes the DFT of the input vector $(0, 2, 3, -1, 4, 5, 7, 9)$.

32.3-2
Show how to implement an FFT algorithm with the bit-reversal permutation occurring at the end, rather than at the beginning, of the computation. (*Hint:* Consider the inverse DFT.)